The multi-path Traveling Salesman Problem with stochastic travel costs

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Roberto Tadei[a,*], Guido Perboli[a,b], Francesca Perfetti[a]

[a] *Politecnico di Torino, Turin, Italy*
[b] *CIRRELT, Montreal, Canada*

Abstract

Given a set of nodes, where each pair of nodes is connected by several paths and each path shows a stochastic travel cost with unknown distribution, the multi-path Traveling Salesman Problem with stochastic travel costs aims at finding an expected minimum Hamiltonian tour connecting all nodes.

The mpTSP$_s$ arises in City Logistics applications when one has to design tours to provide services such as garbage collection, periodic delivery of goods in urban grocery distribution, and periodic checks of shared resources as in bike sharing services. In these situations the decision maker must provide tours that will be used for a time horizon which spans from one to several weeks. In this case the different paths connecting pairs of nodes in the city are affected by the uncertainty due to the different time dependent travel time distributions of the different paths. Moreover, in many cases even an approximated knowledge of the travel time distribution is made difficult by the large size of the data involved and the high variance of the travel times.

This paper introduces the formulation of the mpTSP$_s$. From this formulation a deterministic approximation is derived. In particular, under a mild hypothesis on the unknown probability distribution of the travel time for the different paths, the deterministic approximation becomes a TSP problem where the minimum expected total travel cost is equivalent to the maximum of the logarithm of the total accessibility of the Hamiltonian tours to the path set. The quality of the deterministic approximation is then evaluated by comparing it with the results of a Montecarlo simulation of the stochastic model. The comparison shows a good accuracy of the deterministic approximation, with a mean percentage gap around 2% and a reduction of the computational times of two orders of magnitude.

For references, see [1], [2], [3], [4].

Keywords: TSP, multiple paths, stochastic travel costs, deterministic approximation.

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*Corresponding author:
Department of Control and Computer Engineering, Politecnico di Torino
Corso Duca degli Abruzzi 24, 10129 Torino (Italy)
Tel.: +39 0110907032, Fax: +39 0110907099 E-mail: roberto.tadei@polito.it